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# Hidden Local Symmetry for Anomalous Processes with Isospin/SU(3) Breaking Effects

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## Abstract

We show that isospin/ $SU(3)$  breaking terms can be introduced to the anomalous  $VVP$  coupling in the hidden local symmetry scheme without changing Wess-Zumino-Witten term in the low-energy limit. We make the analysis for anomalous processes of 2-body and 3-body decays; radiative vector meson decays( $V \rightarrow P\gamma$ ), conversion decays of photon into a lepton pair( $V \rightarrow Pl^+l^-$ ) and hadronic anomalous decays( $V \rightarrow PPP$ ). The predictions successfully reproduce all experimental data of anomalous decays. In particular, we predict the decay widths of  $\rho^0 \rightarrow \pi^0\gamma$  and  $\phi \rightarrow \eta'\gamma$  as  $101 \pm 9\text{keV}$  and  $0.508 \pm 0.035\text{keV}$ , respectively, which will be tested in the DAΦNE  $\phi$ -factory. Moreover, prediction is also made for  $\phi \rightarrow \pi^0 e^+e^-$ ,  $\rho \rightarrow 3\pi$ ,  $K^* \rightarrow K\pi\pi$  and so on, for which only the experimental upper bounds are available now.

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## 1 Introduction

Anomalous processes involving vector mesons are interesting probes to test the effective theories of QCD through the low-energy and high-luminosity  $e^+e^-$  collider experiments in near future. In particular, the DAΦNE  $\phi$ -factory is expected to yield  $2 \times 10^{10}$   $\phi$ -meson decays per year [1], which will provide us with high quality data for decays of pseudoscalar( $P$ ) and

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vector mesons( $V$ ) in the light quark sector. It is expected to obtain the branching ratio of  $\phi \rightarrow \eta'\gamma$  [1] for which only the upper bound is known today [2]. Moreover, uncertainty of the data on  $\rho^0 \rightarrow \pi^0\gamma$  will be much reduced [1].

These radiative decays are associated with the flavor anomaly of QCD and are described by the Wess-Zumino-Witten(WZW) term[3] in the low energy limit. Based on the hidden local symmetry(HLS) [4][5][6] for the vector mesons, Fujiwara et al.[7] proposed a systematic way to incorporate vector mesons into such a chiral Lagrangian with WZW term without affecting the low-energy theorem on  $\pi^0 \rightarrow 2\gamma$ ,  $\gamma \rightarrow 3\pi$  etc. Bramon et al.[8] studied extensively the radiative vector meson decays by introducing  $SU(3)$  breaking into the anomalous Lagrangian of Fujiwara et al.[7]. However, the method of Bramon et al. is not consistent with the low-energy theorem, especially on  $\eta(\eta') \rightarrow 2\gamma$ , which are essentially determined by the WZW term. Thus, if isospin breaking effects were introduced through their method, successful low-energy theorem on  $\Gamma(\pi^0 \rightarrow 2\gamma)$  and the coupling of  $\gamma \rightarrow 3\pi$  would be violated. Furthermore, the breaking effects (and  $\rho^0$ - $\omega$  interference effect) are important to account for the difference between  $\Gamma(\rho^0 \rightarrow \pi^0\gamma)$  and  $\Gamma(\rho^\pm \rightarrow \pi^\pm\gamma)$ .

In the previous paper[9], we proposed isospin/ $SU(3)$ -broken anomalous Lagrangians without changing the low energy theorem. These were obtained by eliminating direct  $VP\gamma$  and  $VP^3$  coupled terms, which were absent in the original Lagrangian[7], from all possible isospin/ $SU(3)$ -broken anomalous Lagrangians with the smallest number of derivatives. Then we found a parameter region which was consistent with all the existing data on radiative decays of vector mesons. In this paper, we give a full description of our analysis and  $\chi^2$ -fitting. We also include the analysis of  $V \rightarrow Pl^+l^-$  in addition to the previous results on  $V \rightarrow P\gamma$  and  $V \rightarrow PPP$ .

The paper is organized as follows: In section 2, a review of HLS Lagrangian is given for both non-anomalous and anomalous terms.  $SU(3)$  breaking terms are introduced into the non-anomalous HLS Lagrangian à la Bando et al.[5]. In section 3, we construct the most general isospin/ $SU(3)$ -broken anomalous Lagrangians with the lowest derivatives in a way consistent with the low energy theorem. This is systematically done through spurion method

for the breaking term. In section 4, the phenomenological analysis of these Lagrangians will be successfully done for radiative decays of vector mesons. In section 5, conversion decays of photon into a lepton pair are analyzed. In section 6, we make the analysis for hadronic anomalous decays. Section 7 is devoted to summary and discussions.

## 2 Hidden Local Symmetry

Here we give a brief review of HLS approach[6]. A key observation is that the non-linear sigma model based on the manifold  $U(3)_L \times U(3)_R/U(3)_V$  is gauge equivalent to another model having a symmetry  $[U(3)_L \times U(3)_R]_{\text{global}} \times [U(3)_V]_{\text{local}}$ . Vector mesons are introduced as the gauge fields of a hidden local symmetry  $[U(3)_V]_{\text{local}}$ . The photon field is introduced through gauging a part of  $[U(3)_L \times U(3)_R]_{\text{global}}$ .

The HLS Lagrangian is given by :[4][5]

$$\mathcal{L} = \mathcal{L}_A + a\mathcal{L}_V + \mathcal{L}_{\text{gauge}}, \quad (2.1)$$

$$\mathcal{L}_A = -\frac{f_\pi^2}{8} \text{tr}(D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger)^2, \quad (2.2)$$

$$\mathcal{L}_V = -\frac{f_\pi^2}{8} \text{tr}(D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger)^2, \quad (2.3)$$

where  $f_\pi = 131\text{MeV}$  is the decay constant of pseudoscalar mesons,  $D_\mu \xi_{L,R} \equiv (\partial_\mu - igV_\mu)\xi_{L,R} + ie\xi_{L,R}Q \cdot B_\mu$ , with  $Q = \text{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$ , and with  $V_\mu$  and  $B_\mu$  being the vector mesons and the photon fields, respectively, and  $\mathcal{L}_{\text{gauge}}$  is the kinetic terms of  $V_\mu$  and  $B_\mu$ . We often use an expression  $A_\mu \equiv Q \cdot B_\mu$  as the photon field. Here  $g$ ,  $e$  and  $a$  are respectively the hidden gauge coupling, the electromagnetic coupling and a free parameter not determined by the symmetry considerations alone.

The fields  $\xi_{L,R}$  and  $V_\mu$  transform as follows;

$$\xi_{L,R}(x) \rightarrow \xi'_{L,R}(x) = h(x)\xi_{L,R}(x)g_{L,R}^\dagger(x), \quad (2.4)$$

$$V_\mu(x) \rightarrow V'_\mu(x) = h(x)V_\mu(x)h^\dagger(x) + ih(x)\partial_\mu h^\dagger(x), \quad (2.5)$$

where  $h(x) \in [U(3)_V]_{\text{local}}$ ,  $g_{L,R}(x) \in [U(3)_{L,R}]_{\text{global}}$ . To do a phenomenological analysis, we

take unitary gauge:

$$\xi_R = \xi_L^\dagger = e^{\frac{iP}{f_\pi}}, \quad (2.6)$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}, \quad (2.7)$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \quad (2.8)$$

where we assumed that  $\eta_1$ - $\eta_8$  mixing angle  $\theta_{\eta_1-\eta_8}$  is  $\arcsin(\frac{-1}{3}) \simeq -19.5$  degrees and  $\omega_1$ - $\omega_8$  mixing angle is the ideal mixing (35 degrees). If we take  $a = 2$  in (2.1), we have the celebrated KSRF relation  $M_\rho^2 = 2f_\pi^2 g^2$ , the universality of the  $\rho$ -meson coupling and the vector meson dominance for the electromagnetic form factor(VMD) [4].

For obtaining the pseudoscalar meson mass terms, we introduce the quark mass matrix( $\mathcal{M}$ ) as,

$$\mathcal{L}_M = \frac{f_\pi^2 \mu}{2} \text{tr}(\xi_R \mathcal{M} \xi_L^\dagger + \xi_L \mathcal{M} \xi_R^\dagger) + m_{\eta_1}^2, \quad (2.9)$$

where  $\mu \mathcal{M}$  is related to the mass of  $\pi$ ,  $K$  and  $\eta$ , and  $m_{\eta_1}$  is the mass of  $\eta'$  due to  $U(1)_A$  breaking by the gluon anomaly. Analogously, we may add appropriate  $SU(3)$  breaking terms to (2.1) [5],

$$\Delta \mathcal{L}_{A,(V)} = -\frac{f_\pi^2}{8} \text{tr}(D_\mu \xi_L \cdot \epsilon_{A,(V)} \xi_R^\dagger \pm D_\mu \xi_R \cdot \epsilon_{A,(V)} \xi_L^\dagger)^2, \quad (2.10)$$

$$\epsilon_{A,(V)} = \text{diag}(0, 0, \epsilon_{A,(V)}). \quad (2.11)$$

Even if those  $SU(3)$  breaking terms are introduced, we can show the successful relations [5]:

$$\frac{g_\rho}{M_\rho^2} = \frac{3g_\omega}{M_\omega^2} = -\frac{3g_\phi}{\sqrt{2}M_\phi^2} = \frac{1}{g}. \quad (2.12)$$

We will use this relations, when we consider radiative decays of vector mesons and conversion decays of photon into a lepton pair.

Further improvements of (2.1) have been elaborated in Ref.[10]. Here we will not discuss the non-anomalous sector (2.1) any furthermore, because we are only interested in the

anomalous sector. We simply assume that the parameters of the non-anomalous Lagrangian have been arranged so as to reproduce the relevant experimental data. Thus we use the experimental values as inputs from the non-anomalous part.

In addition to (2.1) there exists an anomalous part of the HLS Lagrangian. Fujiwara et al.[7] proposed how to incorporate vector mesons into this part of the Lagrangian without changing the anomaly determined by WZW term[7]. They have given the anomalous action as follows:

$$\Gamma = \Gamma_{WZW} + \sum_{i=1}^4 \int_{M^4} c_i \mathcal{L}_i, \quad (2.13)$$

where

$$\Gamma_{WZW} = -\frac{iN_c}{240\pi^2} \int_{M^5} \text{tr}[(dU) \cdot U^\dagger]^5_{\text{covariantization}}, \quad (2.14)$$

$$\mathcal{L}_1 = \text{tr}(\hat{\alpha}_L^3 \hat{\alpha}_R - \hat{\alpha}_R^3 \hat{\alpha}_L), \quad (2.15)$$

$$\mathcal{L}_2 = \text{tr}(\hat{\alpha}_L \hat{\alpha}_R \hat{\alpha}_L \hat{\alpha}_R), \quad (2.16)$$

$$\mathcal{L}_3 = i \text{tr} F_V (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L), \quad (2.17)$$

$$\mathcal{L}_4 = \frac{i}{2} \text{tr}(\hat{F}_L + \hat{F}_R) \cdot (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L), \quad (2.18)$$

$$\hat{\alpha}_{L,R} = D\xi_{L,R} \cdot \xi_{L,R}^\dagger = d\xi_{L,R} \cdot \xi_{L,R}^\dagger - igV + ie\xi_{L,R} A \xi_{L,R}^\dagger, \quad (2.19)$$

$$U = \xi_L^\dagger \xi_R, \quad F_V = dV - igV^2, \quad (2.20)$$

$$\hat{F}_{L,R} = \xi_{L,R}(dA - ieA^2)\xi_{L,R}^\dagger. \quad (2.21)$$

Notice that  $\mathcal{L}_1 \sim \mathcal{L}_4$  have no contribution to anomalous processes such as  $\pi^0 \rightarrow 2\gamma$  and  $\gamma \rightarrow 3\pi$  at soft momentum limit, because these Lagrangian are constructed with hidden-gauge covariant blocks such as  $\hat{\alpha}_{L,R}$ ,  $F_V$  and  $\hat{F}_{L,R}$ [7].

We take  $c_3 = c_4 = -15C$ ,  $c_1 - c_2 = 15C$  in (2.13) for phenomenological reason[7]. Then we obtained the Lagrangian of anomalous sector as follows:

$$\begin{aligned} \mathcal{L}_{\text{FKTUY}} &= \frac{-iN_c}{48\pi^2} [3(VVP) - 2(\gamma P^3)] + \dots, \\ (VVP) &= -\frac{2ig^2}{f_\pi} \text{tr}(VdVdP + dVdP), \\ (\gamma P^3) &= \frac{4e}{f_\pi^3} \text{tr}A(dP)^3. \end{aligned} \quad (2.22)$$

Here, it is important that the amplitude such as  $\pi^0 \rightarrow 2\gamma, \gamma \rightarrow 3\pi$  at low energy limit are determined only by the non-Abelian anomaly of the chiral  $U(3)_L \times U(3)_R$  symmetry. The Lagrangian  $\mathcal{L}_{\text{FKTUY}}$  is, of course, consistent with the low energy theorem related to the anomaly.

### 3 Isospin/ $SU(3)$ -breaking Terms in the Anomalous Sector

We now consider how to modify  $\mathcal{L}_1 \sim \mathcal{L}_4$  by introducing isospin/ $SU(3)$ -breaking parameters,  $\epsilon$ 's, treated as “spurions”[11]. The spurion  $\epsilon$  transforms as  $\epsilon \rightarrow g_L \epsilon g_R^\dagger$ . Then we define the hidden-gauge covariant block  $\hat{\epsilon} \equiv \frac{1}{2}(\xi_L \epsilon \xi_R^\dagger + \xi_R \epsilon^\dagger \xi_L^\dagger)$ . We construct Lagrangians out of the hidden-gauge covariant blocks such as  $\hat{\alpha}_{L,R}$ ,  $F_V$ ,  $\hat{F}_{L,R}$  and  $\hat{\epsilon}$  so as to make them “invariant” under  $[U(3)_L \times U(3)_R]_{\text{global}} \times [U(3)_V]_{\text{local}}$  as well as parity( $P$ )-, charge conjugation( $C$ )- and  $CP$ -transformations. After hidden-gauge fixing, they become explicit breaking terms of the  $SU(3)$  symmetry. Then, in general, we obtain isospin/ $SU(3)$ -broken anomalous Lagrangians with the lowest number of derivatives.

$$\Delta\mathcal{L}_1 = \text{tr}[\hat{\alpha}_L^3(\hat{\alpha}_R \cdot \hat{\epsilon}^{(1)} + \hat{\epsilon}^{(1)} \cdot \hat{\alpha}_R) - \hat{\alpha}_R^3(\hat{\alpha}_L \cdot \hat{\epsilon}^{(1)} + \hat{\epsilon}^{(1)} \cdot \hat{\alpha}_L)], \quad (3.1)$$

$$\Delta\mathcal{L}'_1 = \text{tr}(\hat{\alpha}_L \hat{\epsilon}^{(1')} \hat{\alpha}_L^2 \hat{\alpha}_R - \hat{\alpha}_R \hat{\epsilon}^{(1')} \hat{\alpha}_R^2 \hat{\alpha}_L + \hat{\alpha}_L^2 \hat{\epsilon}^{(1')} \hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R^2 \hat{\epsilon}^{(1')} \hat{\alpha}_R \hat{\alpha}_L), \quad (3.2)$$

$$\Delta\mathcal{L}_2 = \text{tr}(\hat{\epsilon}^{(2)} \cdot \hat{\alpha}_L + \hat{\alpha}_L \cdot \hat{\epsilon}^{(2)}) \hat{\alpha}_R \hat{\alpha}_L \hat{\alpha}_R, \quad (3.3)$$

$$\Delta\mathcal{L}_3 = i \text{tr}(F_V \cdot \hat{\epsilon}^{(3)} + \hat{\epsilon}^{(3)} \cdot F_V) \cdot (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L), \quad (3.4)$$

$$\Delta\mathcal{L}'_3 = i \text{tr} F_V (\hat{\alpha}_L \hat{\epsilon}^{(3')} \hat{\alpha}_R - \hat{\alpha}_R \hat{\epsilon}^{(3')} \hat{\alpha}_L), \quad (3.5)$$

$$\Delta\mathcal{L}_4 = i \text{tr}[\{(\hat{F}_L + \hat{F}_R) \cdot \hat{\epsilon}^{(4)} + \hat{\epsilon}^{(4)} \cdot (\hat{F}_L + \hat{F}_R)\} \cdot (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L)], \quad (3.6)$$

$$\Delta\mathcal{L}'_4 = i \text{tr}(\hat{F}_L + \hat{F}_R) \cdot (\hat{\alpha}_L \hat{\epsilon}^{(4')} \hat{\alpha}_R - \hat{\alpha}_R \hat{\epsilon}^{(4')} \hat{\alpha}_L), \quad (3.7)$$

$$\Delta\mathcal{L}_5 = \text{tr} \hat{\epsilon}^{(5)} (\hat{\alpha}_L^2 \hat{\alpha}_R^2 - \hat{\alpha}_R^2 \hat{\alpha}_L^2), \quad (3.8)$$

$$\Delta\mathcal{L}_6 = i \text{tr}(\hat{\epsilon}^{(6)} F_V - F_V \hat{\epsilon}^{(6)}) \cdot (\hat{\alpha}_L^2 - \hat{\alpha}_R^2), \quad (3.9)$$

$$\Delta\mathcal{L}_7 = i \text{tr}[(\hat{\epsilon}^{(7)} \hat{F}_L - \hat{F}_L \hat{\epsilon}^{(7)}) \hat{\alpha}_R^2 - (\hat{\epsilon}^{(7)} \hat{F}_R - \hat{F}_R \hat{\epsilon}^{(7)}) \hat{\alpha}_L^2], \quad (3.10)$$

$$\Delta\mathcal{L}_8 = i \text{tr}[(\hat{\epsilon}^{(8)} \hat{F}_L - \hat{F}_L \hat{\epsilon}^{(8)}) \hat{\alpha}_L^2 - (\hat{\epsilon}^{(8)} \hat{F}_R - \hat{F}_R \hat{\epsilon}^{(8)}) \hat{\alpha}_R^2]. \quad (3.11)$$

Here  $\hat{\alpha}_{L,R}$ ,  $F_V$ ,  $\hat{F}_{L,R}$  transform under  $P$  and  $C$  transformations as

$$P : \quad \hat{\alpha}_{L,R\mu} \longrightarrow \hat{\alpha}_{R,L}^\mu, \quad (3.12)$$

$$F_{V\mu\nu} \longrightarrow F_V^{\mu\nu}, \quad \hat{F}_{L,R\mu\nu} \longrightarrow \hat{F}_{R,L}^{\mu\nu}, \quad (3.13)$$

$$C : \quad \hat{\alpha}_{L,R} \longrightarrow -\hat{\alpha}_{R,L}^T, \quad (3.14)$$

$$F_V \longrightarrow -F_V^T, \quad \hat{F}_{L,R} \longrightarrow -\hat{F}_{R,L}^T. \quad (3.15)$$

We could introduce another “spurion”  $\hat{\epsilon}_- = \frac{1}{2}(\xi_L \epsilon \xi_R^\dagger - \xi_R \epsilon^\dagger \xi_L)$ , which, however, is not relevant to the following analysis.

There still exist too many parameters. However, we may select the combination of  $\Delta\mathcal{L}_{1\sim 8}$  so as to eliminate the direct  $V\gamma P$ -,  $VP^3$ -coupling terms, which do not exist in the original Lagrangian  $\mathcal{L}_{\text{FKTUY}}$ . Then the isospin/ $SU(3)$ -broken anomalous Lagrangians consist of only the following two terms:

$$\begin{aligned} -\Delta\mathcal{L}_{VVP}^a &= \frac{3g^2}{4\pi^2 f_P} \text{tr} \epsilon' (dV dV P + P dV dV) - \frac{3e^2}{4\pi^2 f_P} \text{tr} \epsilon' (dA dA P + P dA dA) \\ &\quad + i \frac{3e}{4\pi^2 f_P^3} \text{tr} \epsilon' (dP^3 A - A dP^3 + dP A dP^2 - dP^2 A dP), \end{aligned} \quad (3.16)$$

$$\begin{aligned} -\Delta\mathcal{L}_{VVP}^b &= \frac{3g^2}{2\pi^2 f_P} \text{tr} \epsilon (dV P dV) - \frac{3e^2}{2\pi^2 f_P} \text{tr} \epsilon (dA P dA) \\ &\quad + i \frac{3e}{2\pi^2 f_P^3} \text{tr} \epsilon (dP^3 A - A dP^3). \end{aligned} \quad (3.17)$$

We can also understand these  $\Delta\mathcal{L}_{VVP}^{a,b}$  in a more straightforward way: We can introduce the breaking terms  $\epsilon$  to the first  $(VVP)$ -term in the original  $\mathcal{L}_{\text{FKTUY}}$  via two possible ways, which correspond to the first terms of  $\Delta\mathcal{L}_{VVP}^{a,b}$ . Next, we determine  $\gamma\gamma P$ -,  $\gamma P^3$ -terms so as to eliminate them at soft momentum limit by using the relation  $gV \rightarrow eA + \frac{i}{2f_\pi^2}[P, dP]$  as well as to make them invariant under  $P$ ,  $C$  and  $CP$ -transformations. These terms correspond to the second and third terms of  $\Delta\mathcal{L}_{VVP}^{a,b}$ .

Our  $\Delta\mathcal{L}_{VVP}^b$  resembles the  $SU(3)$ -broken anomalous Lagrangian introduced by Bramon et al.[8], but is conceptually quite different from the latter. In fact the prediction on  $\eta(\eta') \rightarrow 2\gamma$  decay width in the latter disagrees with the low energy theorem. On the other hand, our  $\Delta\mathcal{L}_{VVP}^{a,b}$  obviously do not change the low energy theorem by construction.

## 4 Phenomenological Analysis for Radiative Decays

We now discuss phenomenological consequences of our Lagrangian  $\mathcal{L}_{\text{anomalous}} = \mathcal{L}_{\text{FKTUY}} + \Delta\mathcal{L}_{VV'P}^a + \Delta\mathcal{L}_{VV'P}^b$ . For convenience, we define relevant coupling constant as

$$g_{VP\gamma} = \sum_{V'} \frac{g_{VV'P}g_{V'}}{M_{V'}^2}, \quad (4.1)$$

considering that these decays proceed via intermediate vector mesons  $V'$ . Then we obtain each radiative decay width

$$\Gamma(V \longrightarrow P\gamma) = \frac{1}{3}\alpha \cdot g_{VP\gamma}^2 \left( \frac{M_V^2 - M_P^2}{2M_V} \right)^3, \quad (4.2)$$

$$\Gamma(\eta' \longrightarrow V\gamma) = \alpha \cdot g_{\eta'V\gamma}^2 \left( \frac{M_{\eta'}^2 - M_V^2}{2M_{\eta'}} \right)^3, \quad (4.3)$$

where  $g_{VV'P}$ ,  $g_{V'}$  and  $M_{V'}$  are anomalous  $VV'P$  coupling constant,  $V'$ - $\gamma$  mixing and mass of the vector meson, respectively. In  $\Delta\mathcal{L}_{VV'P}^{a,b}$  we take a parametrization for convenience:

$$\epsilon' = \begin{pmatrix} -\epsilon'_1 & & \\ & -\epsilon'_2 & \\ & & -\epsilon'_3 \end{pmatrix}, \quad \epsilon = \begin{pmatrix} \epsilon_1 + \epsilon'_1 & & \\ & \epsilon_2 + \epsilon'_2 & \\ & & \epsilon_3 + \epsilon'_3 \end{pmatrix}. \quad (4.4)$$

Thus each  $g_{VP\gamma}$  is given in terms of the parameters in  $\Delta\mathcal{L}_{VV'P}^{a,b}$ :

$$\left\{ \begin{array}{l} g_{\rho^0\pi^0\gamma} = G(1 + 4\epsilon_1 - 2\epsilon_2 + 3\delta), \\ g_{\rho^\pm\pi^\pm\gamma} = G(1 + 3\epsilon'_1 - 3\epsilon'_2 + 4\epsilon_1 - 2\epsilon_2), \\ g_{\omega\pi^0\gamma} = 3G(1 + \frac{4}{3}\epsilon_1 + \frac{2}{3}\epsilon_2 - \frac{\delta}{3}), \\ g_{\omega\eta\gamma} = \frac{f_\pi}{f_{\eta'}}\sqrt{\frac{2}{3}}G(1 + 4\epsilon_1 - 2\epsilon_2 - \sqrt{2}\theta_V - 3\delta - \frac{\theta_P}{\sqrt{2}}), \\ g_{\rho^0\eta\gamma} = \frac{f_\pi}{f_{\eta'}}\sqrt{6}G(1 + \frac{4}{3}\epsilon_1 + \frac{2}{3}\epsilon_2 + \frac{\delta}{3} - \frac{\theta_P}{\sqrt{2}}), \\ g_{\phi\eta\gamma} = \frac{f_\pi}{f_{\eta'}}\frac{2}{\sqrt{3}}G(1 + 2\epsilon_3 + \frac{\theta_V}{\sqrt{2}} + \sqrt{2}\theta_P), \\ g_{K^{*\pm}K^\pm\gamma} = \frac{f_\pi}{f_K}G(1 + 3\epsilon'_1 - 3\epsilon'_3 + 4\epsilon_1 - 2\epsilon_3), \\ g_{K^{*0}K^0\gamma} = -\frac{f_\pi}{f_K}2G(1 + \epsilon_2 + \epsilon_3), \\ g_{\phi\pi^0\gamma} = g_{\omega\pi^0\gamma} \cdot \theta_V, \\ g_{\eta'\rho^0\gamma} = \frac{f_\pi}{f_{\eta'}}\sqrt{3}G(1 + \frac{4}{3}\epsilon_1 + \frac{2}{3}\epsilon_2 + \frac{\delta}{3} + \sqrt{2}\theta_P), \\ g_{\eta'\omega\gamma} = \frac{f_\pi}{f_{\eta'}}\frac{1}{\sqrt{3}}G(1 + 4\epsilon_1 - 2\epsilon_2 + 2\sqrt{2}\theta_V - 3\delta + \sqrt{2}\theta_P), \\ g_{\phi\eta'\gamma} = -\frac{f_\pi}{f_{\eta'}}\frac{2\sqrt{2}}{\sqrt{3}}G(1 + 2\epsilon_3 - \frac{\theta_V}{2\sqrt{2}} - \frac{\theta_P}{\sqrt{2}}), \end{array} \right. \quad (4.5)$$



where  $G = \frac{g}{4\pi^2 f_\pi}$  and we used the relations of (2. 12).

The parameters  $\theta_V, \theta_P$  appearing in the expression of  $g_{VP\gamma}$  stand for the deviation of  $\phi$ - $\omega$ ,  $\eta$ - $\eta'$  mixing angles from the ideal mixing and  $\eta_1$ - $\eta_8$  mixing, respectively. The parameter  $\delta$  comes from the  $\rho$ - $\omega$  interference effect arising from the small mass difference of  $\rho$  and  $\omega$ .

For reproducing the experimental value of  $\Gamma(\phi \rightarrow \rho\pi \rightarrow \pi\pi\pi)$ , we took  $\theta_V = 0.0600 \pm 0.0017$ . The sign comes from the observed  $\phi$ - $\omega$  interference effects in  $e^+e^- \rightarrow \pi^+\pi^-\pi^0$  [2].

The mixing angle  $\theta_{\eta_1-\eta_8} (= \arcsin(-1/3))$  has been supported in  $\eta$ - $\eta'$  phenomenology [13], thus, it is admitted to take  $\theta_P = 0$ .

Similarly, we consider the decay of  $\omega \rightarrow \pi\pi$ , which is  $G$ -parity violating process. If the isospin were not broken, such process would not exist. The experimental value of  $\Gamma(\omega \rightarrow \pi\pi)$  is reproduced for  $\delta = 0.0348 \pm 0.0024$ . We calculated  $\delta$  from  $\Gamma(\omega \rightarrow \pi\pi)/\Gamma(\rho \rightarrow \pi\pi) = \delta^2 \cdot \frac{p_{\omega \rightarrow \pi\pi}^3}{M_\omega^2} / \frac{p_{\rho \rightarrow \pi\pi}^3}{M_\rho^2}$ , where  $p_{V \rightarrow \pi\pi}$  is the final state pion momentum. The ambiguity of the sign has been resolved recently through the decays of  $\omega$  produced in  $\pi^- p \rightarrow \omega n$  [12], in which the constructive interference has been supported.

There are essentially five free parameters from  $\Delta\mathcal{L}_{VVP}^{a,b}$  in (4. 5), because  $\epsilon'_1$  is negligible. We determine these parameters by  $\chi^2$  fitting, using the data of the radiative vector meson decays and  $\omega \rightarrow 3\pi$ . Then we obtain  $\epsilon_1 = -0.0174 \pm 0.0100$ ,  $\epsilon_2 = -0.0246 \pm 0.0114$ ,  $\epsilon_3 = -0.0974 \pm 0.0103$ ,  $\epsilon'_1 - \epsilon'_2 = -0.0292 \pm 0.0015$  and  $\epsilon'_1 - \epsilon'_3 = 0.0366 \pm 0.0028$ .

We take  $g = 4.27 \pm 0.02$  from  $\Gamma(\rho \rightarrow \pi\pi) = 151.2 \pm 1.2\text{MeV}$ , and  $f_\pi = 131\text{MeV}$ ,  $f_K = 160 \pm 2\text{MeV}$  [2], and  $f_\eta = 150 \pm 6\text{MeV}$ ,  $f_{\eta'} = 142 \pm 3\text{MeV}$  from  $\eta(\eta') \rightarrow 2\gamma$  [2]. Then we obtained the results listed in Table I.

In Table I, (i)~(iii) mean:

- $$\left\{ \begin{array}{l} \text{(i) Values of original } \mathcal{L}_{\text{FKTUY}}. \\ \text{(ii) Values of the } SU(3) \text{ -broken model by Bramon et al. [8] } (\epsilon_3 = -0.1 \pm 0.03). \\ \text{(iii) Values of our model.} \\ \quad \text{The parameter values are } \epsilon_1 = -0.0174 \pm 0.0100, \epsilon_2 = -0.0246 \pm 0.0114, \\ \quad \epsilon_3 = -0.0974 \pm 0.0103, \epsilon'_1 - \epsilon'_2 = -0.0292 \pm 0.0015 \text{ and } \epsilon'_1 - \epsilon'_3 = 0.0366 \pm 0.0028. \end{array} \right.$$

These parameters suggest that isospin/ $SU(3)$ -breaking effects for the anomalous sector cannot be given by the quark mass matrix in a simple manner. We discuss this point later.

In the previous paper[9], we determined a parameter region so as to reproduce all experimental data of radiative vector meson decays as  $0.0279 < 4\epsilon_1 - 2\epsilon_2 < 0.0670$ ,  $-0.0471 < \frac{4}{3}\epsilon_1 + \frac{2}{3}\epsilon_2 < -0.0174$ ,  $-0.0112 < \epsilon_2 + \epsilon_3 < -0.0902$ ,  $-0.0925 < \epsilon_3 < -0.0702$ ,  $4\epsilon_1 - 2\epsilon_2 + 3\epsilon'_1 - 3\epsilon'_2 = -0.108$  and  $4\epsilon_1 - 2\epsilon_3 + 3\epsilon'_1 - 3\epsilon'_3 = 0.235$ . The parameters by  $\chi^2$ -fitting are slightly different from the above region, which then yield the prediction of  $\Gamma(\omega \rightarrow \eta\gamma)$  from the experimental value. However, the difference of the former maximum value from the latter minimum value is about 5%. If we consider that the experimental data[2] is determined only by Ref.[12], where large momentum transfer events have been selected in order to eliminate  $\rho$ - $\omega$  interference contribution, our result is not inconsistent with the experiments. In fact, if we take the experimental value of Ref.[14], where the decay width of  $\omega \rightarrow \eta\gamma$  has been reported as  $6.2 \pm 2.4 \text{ keV}$  based on the assumption of constructive  $\rho$ - $\omega$  interference, our prediction by  $\chi^2$ -fitting also reproduces the experimental value.

The results for  $\Gamma(\rho^0 \rightarrow \pi^0\gamma)$ ,  $\Gamma(\rho^\pm \rightarrow \pi^\pm\gamma)$  and  $\Gamma(\omega \rightarrow \pi^0\gamma)$  in Table I suggest that isospin breaking terms are very important. Both (i) and (ii) in Table I do not have isospin breaking terms. These values differ substantially from the experiments, which cannot be absorbed by the ambiguity of the hidden-gauge coupling  $g$  whose value is determined either by  $\Gamma(\rho \rightarrow 2\pi)$  or by  $\Gamma(\rho \rightarrow e^+e^-)$ . In order to avoid this ambiguity, let us take some expressions cancelling  $g$ , i.e. ,  $\Gamma(\rho \rightarrow \pi\gamma)/\Gamma(\rho \rightarrow 2\pi)$  and  $\Gamma(\omega \rightarrow \pi\gamma)/\Gamma(\rho \rightarrow 2\pi)$ . Then we find that predictions of the original  $\mathcal{L}_{\text{FKTUY}}$  and Bramer et al.[8] still do not agree with the experiments. These Lagrangians without isospin breaking terms yield

$$\begin{aligned} \frac{\Gamma(\rho^0 \rightarrow \pi^0\gamma)}{\Gamma(\rho \rightarrow 2\pi)} &= \alpha M_\rho^2 p_{\rho \rightarrow \pi\gamma}^3 / 16\pi^3 f_\pi^2 p_{\rho \rightarrow \pi\pi}^3, \\ &= 5.7 \times 10^{-4} \text{ [from (i) and (ii)],} \\ &[(6.6 \pm 0.6) \times 10^{-4} \text{ (iii) Ours }], \\ &[(7.9 \pm 2.0) \times 10^{-4} \text{ exp.}], \end{aligned} \tag{4.6}$$

$$\begin{aligned} \frac{\Gamma(\rho^\pm \rightarrow \pi^\pm\gamma)}{\Gamma(\rho \rightarrow 2\pi)} &= \alpha M_\rho^2 p_{\rho \rightarrow \pi\gamma}^3 / 16\pi^3 f_\pi^2 p_{\rho \rightarrow \pi\pi}^3, \\ &= 5.7 \times 10^{-4} \text{ [from (i) and (ii)],} \\ &[(4.5 \pm 0.5) \times 10^{-4} \text{ (iii) Ours }], \end{aligned} \tag{4.7}$$

$$\begin{aligned}
& [(4.5 \pm 0.5) \times 10^{-4} \text{ exp.}] , \\
\frac{\Gamma(\omega \rightarrow \pi^0 \gamma)}{\Gamma(\rho \rightarrow 2\pi)} &= 9\alpha M_\rho^2 p_{\rho \rightarrow \pi\gamma}^3 / 16\pi^3 f_\pi^2 p_{\rho \rightarrow \pi\pi}^3 , \\
&= 5.4 \times 10^{-3} [\text{from (i) and (ii)}], \\
& [(4.9 \pm 0.2) \times 10^{-3} \text{ (iii) Ours } ] , \\
& [(4.7 \pm 0.4) \times 10^{-3} \text{ exp. } ] .
\end{aligned} \tag{4.8}$$

Finally, we pay attention to  $\Gamma(\eta(\eta') \rightarrow \pi^+ \pi^- \gamma)$ , which are given by

$$\begin{aligned}
\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma) &= \frac{3g^2 \alpha}{16\pi^6 f_\eta^2 M_\eta} \int dE_+ dE_- [\mathbf{p}_+^2 \mathbf{p}_-^2 - (\mathbf{p}_+ \cdot \mathbf{p}_-)^2] \times \\
&\quad \left( \frac{1 + 4/3\epsilon_1 + 2/3\epsilon_2}{(p_+ + p_-)^2 - M_\rho^2} + \frac{1 + 4\epsilon_1 + 2\epsilon_2}{3M_\rho^2} \right)^2 ,
\end{aligned} \tag{4.9}$$

$$\begin{aligned}
\Gamma(\eta' \rightarrow \pi^+ \pi^- \gamma) &= \frac{3g^2 \alpha}{32\pi^6 f_{\eta'}^2 M_{\eta'}} \int dE_+ dE_- [\mathbf{p}_+^2 \mathbf{p}_-^2 - (\mathbf{p}_+ \cdot \mathbf{p}_-)^2] \times \\
&\quad \left( \frac{1 + 4/3\epsilon_1 + 2/3\epsilon_2}{(p_+ + p_-)^2 + iM_\rho \Gamma_\rho - M_\rho^2} + \frac{1 + 4\epsilon_1 + 2\epsilon_2}{3M_\rho^2} \right)^2 ,
\end{aligned} \tag{4.10}$$

$$\Gamma_\rho = \Gamma(\rho \rightarrow 2\pi) \cdot \left( \frac{(p_+ + p_-)^2 - 4M_\pi^2}{M_\rho^2 - 4M_\pi^2} \right)^{3/2} \theta((p_+ + p_-)^2 - 4M_\pi^2) ,$$

where  $p_\pm$  are pion momenta and we express  $\rho$ -meson propagator in the process  $\eta' \rightarrow \rho^0 \gamma \rightarrow \pi^+ \pi^- \gamma$  by using the  $\rho$ -meson decay width  $\Gamma_\rho$ .

One might wonder why the prediction of  $\Gamma(\eta(\eta') \rightarrow \pi^+ \pi^- \gamma)$  by our model is different from the prediction by the original  $\mathcal{L}_{\text{FKTUY}}$  in Table I, because each model does not change the low-energy theorem. The answer to the question is that  $p_\pm$  in  $\rho$ -meson propagator are not soft momenta in the decay of  $\eta(\eta') \rightarrow \pi^+ \pi^- \gamma$ . If we take the low-energy limit in (4.9) and (4.10), we find that isospin/ $SU(3)$  breaking effects are cancelled, and each model is actually consistent with the low-energy theorem.

## 5 The conversion decays of photon into a lepton pair

In this section, we make the analysis for the decays of  $V \rightarrow Pl^+l^-$ . Each decay width is given by

$$\Gamma(V \rightarrow Pl^+l^-) = \frac{\alpha^2}{48\pi M_V} \int dx_+ dx_- T \times \left( \sum_{V'} \frac{g_{VV'P}}{(x_+ + x_- - 1 + \mu_P - \mu_{V'})} \frac{g_{V'}}{(x_+ + x_- - 1 + \mu_P)} \right)^2, \quad (5.1)$$

$$\begin{aligned} T &\equiv (1 - \mu_P - x_+)^2 \cdot (x_+ + x_- - 1 + \mu_P) + (1 - \mu_P - x_-)^2 \cdot (x_+ + x_- - 1 + \mu_P) \\ &\quad - 2\mu_P(x_+ + x_- - 1 + \mu_P)^2 - 2\mu_l \left[ 4\mu_P(x_+ + x_- - 1 + \mu_P) - (2 - 2\mu_P - x_+ - x_-)^2 \right], \\ x_{\pm} &\equiv \frac{2E_{l^{\pm}}}{M_V}, \quad \mu_{V',P,l} \equiv \frac{M_{V',P,l}^2}{M_V^2}. \end{aligned}$$

Then we obtain Table II.

In conversion decays, we can also show importance of the isospin breaking effects on  $\omega \rightarrow \pi^0 e^+ e^-$ . As in the previous section, we take an expression eliminating the hidden gauge coupling  $g$ :

$$\begin{aligned} \frac{\Gamma(\omega \rightarrow \pi^0 e^+ e^-)}{\Gamma(\rho \rightarrow \pi\pi)} &= 4.91 \times 10^{-5} [\text{original } \mathcal{L}_{\text{FKTU}}], \\ &[(4.42 \pm 0.13) \times 10^{-5} \text{ Ours}], \\ &[(3.29 \pm 1.06) \times 10^{-5} \text{ exp.}]. \end{aligned}$$

Our model reproduces successfully the experimental value of  $\Gamma(\omega \rightarrow \pi^0 e^+ e^-)$  without such a trick as in the lattice results of Crisafulli et al.[1], who actually made ansatz of linearized expression in  $p^2$  of the lepton momentum for VMD form factor, assuming  $p^2$  is small. We disagree with the lattice results because  $p$  can be the same order as  $\omega$  mass(=782MeV), which is not small.

## 6 Hadronic Anomalous Decays

In this section, we consider hadronic anomalous decays such as  $\Gamma(\omega \rightarrow 3\pi)$ . In the same way as in the previous section, we obtained Table III.

In Table III, we took  $K^*K\pi$ -coupling as  $1.05 \times g_{K^*K\pi}$ , which is given by (2. 1), considering  $\Gamma(K^{*\pm} \rightarrow (K\pi)^\pm) = 49.8 \pm 0.8$  MeV,  $\Gamma(K^{*0} \rightarrow (K\pi)^0) = 50.5 \pm 0.6$  MeV.

In Table III it is again suggested that isospin breaking terms are very important. As in the previous section, let us take an expression cancelling  $g$ , i.e. ,  $\Gamma(\omega \rightarrow 3\pi)/\Gamma(\rho \rightarrow 2\pi)^3$ . Then we find that the prediction of the original  $\mathcal{L}_{\text{FKTUY}}$  is again different from the experiments:

$$\begin{aligned}
\frac{\Gamma(\omega \rightarrow 3\pi)}{\Gamma(\rho \rightarrow 2\pi)^3} &= \frac{81M_\omega M_\rho^6(1 + \epsilon_1 + \epsilon_2)^2}{256\pi^4 f_\pi^2 p_{\rho \rightarrow \pi\pi}^9} \int dE_+ dE_- [\mathbf{p}_+^2 \mathbf{p}_-^2 - (\mathbf{p}_+ \cdot \mathbf{p}_-)^2] \times \\
&\quad \left( \frac{1}{(p_0 + p_+)^2 + M_\rho^2} + \frac{1}{(p_+ + p_-)^2 + M_\rho^2} + \frac{1}{(p_- + p_0)^2 + M_\rho^2} \right)^2 \quad (6. 1) \\
&= 2.38 \times 10^{-6} \text{ MeV}^{-2} \text{ [ original } \mathcal{L}_{\text{FKTUY}} \text{ with } \epsilon_1 = \epsilon_2 = 0 \text{ ]}, \\
&\quad [ (2.17 \pm 0.07) \times 10^{-6} \text{ MeV}^{-2} \text{ Ours } ], \\
&\quad [ (2.16 \pm 0.09) \times 10^{-6} \text{ MeV}^{-2} \text{ exp. } ].
\end{aligned}$$

Although only the upper bounds of  $\Gamma(\rho \rightarrow 3\pi)$  and  $\Gamma(K^* \rightarrow K\pi\pi)$  are available now, it is interesting that their value will be determined by the experiments in future.

## 7 Summary and Discussions

By introducing isospin/ $SU(3)$ -broken  $\Delta\mathcal{L}_{VV P}^{a,b}$  with a few parameters, we have shown that decay widths of anomalous processes can be reproduced consistently with all the experimental data.

We now discuss the origin of the isospin/ $SU(3)$  breaking parameters. From (4. 4), we obtain the isospin/ $SU(3)$ -breaking parameters as follows:

$$\epsilon' = \begin{pmatrix} -\epsilon'_1 & & \\ & -0.0292 - \epsilon'_1 & \\ & & 0.0366 - \epsilon'_1 \end{pmatrix}, \quad (7. 1)$$

$$\epsilon = \begin{pmatrix} -0.0174 + \epsilon'_1 & & \\ & 0.0047 + \epsilon'_1 & \\ & & 0.1339 + \epsilon'_1 \end{pmatrix}, \quad (7. 2)$$

where  $\epsilon'_1$  is an arbitrary parameter. If we adopt quark mass matrix as isospin/ $SU(3)$ -breaking terms in a usual manner,  $\epsilon'$  of (7. 1) must be proportional to  $\epsilon$  of (7. 2). On this condition, however, we find easily that this strategy is absolutely inconsistent, even if we consider the error bar of these parameters. Thus it seems to need further discussions on the origin of these parameters. We may suggest that these parameters are deduced from loop effects of vector mesons to anomalous  $VVP$ -couplings. In this case, we inevitably need to consider the effects of  $(VP^3)$ -terms from  $\Delta\mathcal{L}_{1,2,5}$ . However, their contributions seem to be very small compared with the contributions from  $\Delta\mathcal{L}_{VVP}^{a,b}$ , because our prediction of  $\Gamma(\omega \rightarrow 3\pi)$  is already consistent with the experimental value.

Our predictions for future experiments given in Table I, Table II and Table III, are summarized in Table IV.

We expect that the decay data for pseudoscalar mesons and vector mesons, such as  $\phi \rightarrow \eta'\gamma, \rho^0 \rightarrow \pi^0\gamma, \phi \rightarrow \pi^0 e^+ e^-$  and so on, will be obtained with good accuracy in the DAΦNE  $\phi$ -factory.

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Table 1: Radiative Decay Width of Vector Mesons

(i) Values of original  $\mathcal{L}_{\text{FKTUY}}$  [7]

(ii) Values of the  $SU(3)$ -broken model by Bramon et al.[8] ( $\epsilon_3 = -0.1 \pm 0.03$ ).

(iii) Values of our model by  $\chi^2$ -fitting. The parameters are

$\epsilon_1 = -0.0174 \pm 0.0100$ ,  $\epsilon_2 = -0.0246 \pm 0.0114$ ,  $\epsilon_3 = -0.0974 \pm 0.0103$ ,

$\epsilon'_1 - \epsilon'_2 = -0.0292 \pm 0.0015$  and  $\epsilon'_1 - \epsilon'_3 = 0.0366 \pm 0.0028$ .

Decay Mode	(i) $\mathcal{L}_{\text{FKTUY}}$ [7]	(ii) Bramon's [8]	(iii) Ours	exp.[2]
$\Gamma(\rho^0 \rightarrow \pi^0 \gamma)$	$86.2 \pm 0.8\text{keV}$	$86.2 \pm 0.8\text{keV}$	$101 \pm 9\text{keV}$	$121 \pm 31\text{keV}$
$\Gamma(\rho^\pm \rightarrow \pi^\pm \gamma)$	$85.6 \pm 0.8\text{keV}$	$85.6 \pm 0.8\text{keV}$	$68.1 \pm 7.1\text{keV}$	$68 \pm 7\text{keV}$
$\Gamma(\omega \rightarrow \pi^0 \gamma)$	$815 \pm 8\text{keV}$	$815 \pm 8\text{keV}$	$734 \pm 21\text{keV}$	$717 \pm 43\text{keV}$
$\Gamma(\omega \rightarrow \eta \gamma)$	$6.68 \pm 0.59\text{keV}$	$5.6 \pm 0.6\text{keV}$	$4.17 \pm 0.77\text{keV}$	$7.00 \pm 1.77\text{keV}$
$\Gamma(\rho^0 \rightarrow \eta \gamma)$	$52.4 \pm 4.6\text{keV}$	$52.4 \pm 4.6\text{keV}$	$49.5 \pm 4.9\text{keV}$	$57.5 \pm 10.6\text{keV}$
$\Gamma(\phi \rightarrow \eta \gamma)$	$80.7 \pm 7.1\text{keV}$	$57 \pm 9\text{keV}$	$58.0 \pm 7.6\text{keV}$	$56.9 \pm 2.9\text{keV}$
$\Gamma(K^{*\pm} \rightarrow K^\pm \gamma)$	$32.8 \pm 0.9\text{keV}$	$47 \pm 5\text{keV}$	$50.0 \pm 3.9\text{keV}$	$50 \pm 5\text{keV}$
$\Gamma(K^{*0} \rightarrow \bar{K}^0 \gamma)$	$132 \pm 4\text{keV}$	$107 \pm 15\text{keV}$	$102 \pm 5\text{keV}$	$117 \pm 10\text{keV}$
$\Gamma(\phi \rightarrow \pi^0 \gamma)$	— — —	$6.76 \pm 0.34\text{keV}$	$6.09 \pm 0.39\text{keV}$	$5.80 \pm 0.58\text{keV}$
$\Gamma(\eta' \rightarrow \rho^0 \gamma)$	$61.7 \pm 2.7\text{keV}$	$61.7 \pm 2.7\text{keV}$	$58.3 \pm 3.3\text{keV}$	$61 \pm 5\text{keV}$
$\Gamma(\eta' \rightarrow \omega \gamma)$	$5.74 \pm 0.25\text{keV}$	$7.86 \pm 0.34\text{keV}$	$6.27 \pm 0.61\text{keV}$	$6.1 \pm 0.8\text{keV}$
$\Gamma(\phi \rightarrow \eta' \gamma)$	$0.827 \pm 0.036\text{keV}$	$0.5 \pm 0.1\text{keV}$	$0.508 \pm 0.035\text{keV}$	$< 1.84\text{keV}$
$\Gamma(\pi^0 \rightarrow 2\gamma)$	$7.70\text{eV}$	$7.70\text{eV}$	$7.70\text{eV}$	$7.7 \pm 0.6\text{eV}$
$\Gamma(\eta \rightarrow 2\gamma)$	$0.46 \pm 0.04\text{keV}$	$0.51 \pm 0.04\text{keV}$	$0.46 \pm 0.04\text{keV}$	$0.46 \pm 0.04\text{keV}$
$\Gamma(\eta' \rightarrow 2\gamma)$	$4.26 \pm 0.19\text{keV}$	$3.6 \pm 0.2\text{keV}$	$4.26 \pm 0.19\text{keV}$	$4.26 \pm 0.19\text{keV}$
$\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma)$	$0.0660 \pm 0.0053\text{keV}$	$0.0660 \pm 0.0053\text{keV}$	$0.0641 \pm 0.0054\text{keV}$	$0.0586 \pm 0.0057\text{keV}$
$\Gamma(\eta' \rightarrow \pi^+ \pi^- \gamma)$	$53.0 \pm 2.2\text{keV}$	$53.0 \pm 2.2\text{keV}$	$49.5 \pm 2.1\text{keV}$	$56.1 \pm 6.4\text{keV}$



Table 2: Decay Width of  $V \rightarrow Pl^+l^-$

In our model, the parameters are

$$\epsilon_1 = -0.0174 \pm 0.0100, \epsilon_2 = -0.0246 \pm 0.0114, \epsilon_3 = -0.0974 \pm 0.0103,$$

$$\epsilon'_1 - \epsilon'_2 = -0.0292 \pm 0.0015 \text{ and } \epsilon'_1 - \epsilon'_3 = 0.0366 \pm 0.0028.$$

Decay Mode	$\mathcal{L}_{\text{FKTUY}}$ [7]	Ours	exp.[2]
$\Gamma(\rho^0 \rightarrow \pi^0 e^+ e^-)$	$0.778 \pm 0.007 \text{keV}$	$0.914 \pm 0.079 \text{keV}$	— — — — —
$\Gamma(\rho^0 \rightarrow \pi^0 \mu^+ \mu^-)$	$0.0734 \pm 0.0007 \text{keV}$	$0.0863 \pm 0.0075 \text{keV}$	— — — — —
$\Gamma(\rho^\pm \rightarrow \pi^\pm e^+ e^-)$	$0.771 \pm 0.007 \text{keV}$	$0.614 \pm 0.065 \text{keV}$	— — — — —
$\Gamma(\rho^\pm \rightarrow \pi^\pm \mu^+ \mu^-)$	$0.0719 \pm 0.0007 \text{keV}$	$0.0572 \pm 0.0060 \text{keV}$	— — — — —
$\Gamma(\omega \rightarrow \pi^0 e^+ e^-)$	$7.42 \pm 0.07 \text{keV}$	$6.68 \pm 0.22 \text{keV}$	$4.97 \pm 1.60 \text{keV}$
$\Gamma(\omega \rightarrow \pi^0 \mu^+ \mu^-)$	$0.745 \pm 0.007 \text{keV}$	$0.670 \pm 0.022 \text{keV}$	$0.809 \pm 0.194 \text{keV}$
$\Gamma(\rho^0 \rightarrow \eta e^+ e^-)$	$0.377 \pm 0.031 \text{keV}$	$0.301 \pm 0.039 \text{keV}$	— — — — —
$\Gamma(\rho^0 \rightarrow \eta \mu^+ \mu^-)$	$(1.40 \pm 0.11) \times 10^{-5} \text{keV}$	$(1.12 \pm 0.15) \times 10^{-5} \text{keV}$	— — — — —
$\Gamma(\omega \rightarrow \eta e^+ e^-)$	$0.0487 \pm 0.0039 \text{keV}$	$0.0304 \pm 0.0043 \text{keV}$	— — — — —
$\Gamma(\omega \rightarrow \eta \mu^+ \mu^-)$	$(1.36 \pm 0.11) \times 10^{-5} \text{keV}$	$(0.850 \pm 0.012) \times 10^{-5} \text{keV}$	— — — — —
$\Gamma(\phi \rightarrow \eta e^+ e^-)$	$0.677 \pm 0.055 \text{keV}$	$0.486 \pm 0.046 \text{keV}$	$0.576^{+0.354}_{-0.266} \text{keV}$
$\Gamma(\phi \rightarrow \eta \mu^+ \mu^-)$	$0.0325 \pm 0.0026 \text{keV}$	$0.0233 \pm 0.0022 \text{keV}$	— — — — —
$\Gamma(K^{*\pm} \rightarrow K^\pm e^+ e^-)$	$0.272 \pm 0.007 \text{keV}$	$0.414 \pm 0.032 \text{keV}$	— — — — —
$\Gamma(K^{*\pm} \rightarrow K^\pm \mu^+ \mu^-)$	$0.00962 \pm 0.00026 \text{keV}$	$0.0141 \pm 0.0011 \text{keV}$	— — — — —
$\Gamma(\bar{K}^{*0} \rightarrow \bar{K}^0 e^+ e^-)$	$4.32 \pm 0.12 \text{keV}$	$3.34 \pm 0.15 \text{keV}$	— — — — —
$\Gamma(\bar{K}^{*0} \rightarrow \bar{K}^0 \mu^+ \mu^-)$	$0.131 \pm 0.003 \text{keV}$	$0.102 \pm 0.004 \text{keV}$	— — — — —
$\Gamma(\phi \rightarrow \pi^0 e^+ e^-)$	— — — — —	$0.0640 \pm 0.0042 \text{keV}$	$< 0.532 \text{keV}$
$\Gamma(\phi \rightarrow \pi^0 \mu^+ \mu^-)$	— — — — —	$0.0137 \pm 0.0009 \text{keV}$	— — — — —
$\Gamma(\eta' \rightarrow \rho^0 e^+ e^-)$	$0.428 \pm 0.019 \text{keV}$	$0.341 \pm 0.024 \text{keV}$	— — — — —
$\Gamma(\eta' \rightarrow \omega e^+ e^-)$	$0.0392 \pm 0.0017 \text{keV}$	$0.0245 \pm 0.0031 \text{keV}$	— — — — —
$\Gamma(\phi \rightarrow \eta' e^+ e^-)$	$0.00407 \pm 0.00018 \text{keV}$	$0.00262 \pm 0.00018 \text{keV}$	— — — — —
$\Gamma(\pi^0 \rightarrow \gamma e^+ e^-)$	$0.0920 \text{eV}$	$0.0920 \pm 0.0001 \text{eV}$	$0.0922 \pm 0.0076 \text{eV}$
$\Gamma(\eta \rightarrow \gamma e^+ e^-)$	$7.79 \pm 0.62 \text{eV}$	$7.78 \pm 0.62 \text{eV}$	$6.00 \pm 1.54 \text{eV}$
$\Gamma(\eta \rightarrow \gamma \mu^+ \mu^-)$	$0.373 \pm 0.030 \text{eV}$	$0.370 \pm 0.031 \text{eV}$	$0.372 \pm 0.059 \text{eV}$
$\Gamma(\eta' \rightarrow \gamma e^+ e^-)$	$0.0859 \pm 0.0036 \text{keV}$	$0.0852 \pm 0.0037 \text{keV}$	— — — — —
$\Gamma(\eta' \rightarrow \gamma \mu^+ \mu^-)$	$0.0192 \pm 0.0008 \text{keV}$	$0.0183 \pm 0.0014 \text{keV}$	$0.0209 \pm 0.0055 \text{keV}$

Table 3: Hadronic Decay Width of Vector Mesons

In our model, the parameters are

$\epsilon_1 = -0.0174 \pm 0.0100$ ,  $\epsilon_2 = -0.0246 \pm 0.0114$ ,  $\epsilon_3 = -0.0974 \pm 0.0103$ ,  
 $\epsilon'_1 - \epsilon'_2 = -0.0292 \pm 0.0015$  and  $\epsilon'_1 - \epsilon'_3 = 0.0366 \pm 0.0028$ .

Decay Mode	$\mathcal{L}_{\text{FKTUY}}$ [7]	Ours	exp. [2]
$\Gamma(\omega \rightarrow \pi^0 \pi^+ \pi^-)$	$8.18 \pm 0.23 \text{MeV}$	$7.51 \pm 0.33 \text{MeV}$	$7.49 \pm 0.12 \text{MeV}$
$\Gamma(\rho^0 \rightarrow \pi^0 \pi^+ \pi^-)$	— — — —	$8.12 \pm 2.35 \text{keV}$	$< 18 \text{keV}$
$\Gamma(\rho^\pm \rightarrow \pi^\pm \pi^0 \pi^0)$	— — — —	$2.11 \pm 0.43 \text{keV}$	— — — —
$\Gamma(\rho^\pm \rightarrow \pi^\pm \pi^+ \pi^-)$	— — — —	$0.141 \pm 0.071 \text{keV}$	— — — —
$\Gamma(K^{*-} \rightarrow \bar{K}^0 \pi^0 \pi^-)$	$17.9 \pm 0.7 \text{keV}$	$11.5 \pm 0.5 \text{keV}$	$< 35 \text{keV}$
$\Gamma(K^{*-} \rightarrow K^- \pi^+ \pi^-)$	$8.65 \pm 0.33 \text{keV}$	$5.83 \pm 0.23 \text{keV}$	$< 40 \text{keV}$
$\Gamma(K^{*-} \rightarrow K^- \pi^0 \pi^0)$	$1.11 \pm 0.04 \text{keV}$	$0.593 \pm 0.025 \text{keV}$	— — — —
$\Gamma(\bar{K}^{*0} \rightarrow K^- \pi^0 \pi^+)$	$23.2 \pm 0.9 \text{keV}$	$15.9 \pm 0.6 \text{keV}$	— — — —
$\Gamma(\bar{K}^{*0} \rightarrow \bar{K}^0 \pi^- \pi^+)$	$9.04 \pm 0.34 \text{keV}$	$5.94 \pm 0.23 \text{keV}$	$< 35 \text{keV}$
$\Gamma(\bar{K}^{*0} \rightarrow \bar{K}^0 \pi^0 \pi^0)$	$1.11 \pm 0.05 \text{keV}$	$0.507 \pm 0.022 \text{keV}$	— — — —

Table 4: The List of Our Predictions

Decay Mode	Decay Width [keV]	B.R.
$\Gamma(\rho^0 \rightarrow \pi^0 \gamma)$	$101 \pm 9$	$(6.68 \pm 0.58) \times 10^{-4}$
$\Gamma(\phi \rightarrow \eta' \gamma)$	$0.508 \pm 0.035$	$(1.15 \pm 0.08) \times 10^{-4}$
$\Gamma(\rho^0 \rightarrow \pi^0 e^+ e^-)$	$0.914 \pm 0.079$	$(6.08 \pm 0.53) \times 10^{-6}$
$\Gamma(\rho^0 \rightarrow \pi^0 \mu^+ \mu^-)$	$0.0863 \pm 0.0075$	$(5.71 \pm 0.50) \times 10^{-7}$
$\Gamma(\rho^\pm \rightarrow \pi^\pm e^+ e^-)$	$0.614 \pm 0.065$	$(4.61 \pm 0.43) \times 10^{-6}$
$\Gamma(\rho^\pm \rightarrow \pi^\pm \mu^+ \mu^-)$	$0.0572 \pm 0.0060$	$(3.78 \pm 0.40) \times 10^{-7}$
$\Gamma(\rho^0 \rightarrow \eta e^+ e^-)$	$0.301 \pm 0.039$	$(1.99 \pm 0.26) \times 10^{-6}$
$\Gamma(\omega \rightarrow \eta e^+ e^-)$	$0.0304 \pm 0.0043$	$(3.61 \pm 0.51) \times 10^{-6}$
$\Gamma(\phi \rightarrow \eta e^+ e^-)$	$0.486 \pm 0.046$	$1.10 \pm 0.10 \times 10^{-4}$
$\Gamma(\phi \rightarrow \eta \mu^+ \mu^-)$	$0.0233 \pm 0.0022$	$(5.26 \pm 0.50) \times 10^{-6}$
$\Gamma(K^{*\pm} \rightarrow K^\pm e^+ e^-)$	$0.414 \pm 0.032$	$(8.31 \pm 0.66) \times 10^{-6}$
$\Gamma(K^{*\pm} \rightarrow K^\pm \mu^+ \mu^-)$	$0.0141 \pm 0.0011$	$(2.83 \pm 0.23) \times 10^{-7}$
$\Gamma(\bar{K}^{*0} \rightarrow \bar{K}^0 e^+ e^-)$	$3.34 \pm 0.15$	$(6.61 \pm 0.31) \times 10^{-5}$
$\Gamma(\bar{K}^{*0} \rightarrow \bar{K}^0 \mu^+ \mu^-)$	$0.102 \pm 0.004$	$(2.02 \pm 0.08) \times 10^{-6}$
$\Gamma(\phi \rightarrow \pi^0 e^+ e^-)$	$0.0640 \pm 0.0042$	$(1.44 \pm 0.10) \times 10^{-5}$
$\Gamma(\phi \rightarrow \pi^0 \mu^+ \mu^-)$	$0.0137 \pm 0.0009$	$(3.09 \pm 0.21) \times 10^{-6}$
$\Gamma(\eta' \rightarrow \rho^0 e^+ e^-)$	$0.341 \pm 0.024$	$1.70 \pm 0.18 \times 10^{-3}$
$\Gamma(\eta' \rightarrow \omega e^+ e^-)$	$0.0245 \pm 0.0031$	$(1.22 \pm 0.18) \times 10^{-4}$
$\Gamma(\phi \rightarrow \eta' e^+ e^-)$	$0.00262 \pm 0.00018$	$(5.91 \pm 0.41) \times 10^{-7}$
$\Gamma(\eta' \rightarrow \gamma e^+ e^-)$	$0.0852 \pm 0.0037$	$(4.24 \pm 0.38) \times 10^{-4}$
$\Gamma(\rho^0 \rightarrow \pi^0 \pi^+ \pi^-)$	$8.12 \pm 2.35$	$(5.37 \pm 1.56) \times 10^{-5}$
$\Gamma(\rho^\pm \rightarrow \pi^\pm \pi^0 \pi^0)$	$2.11 \pm 0.43$	$(1.40 \pm 0.28) \times 10^{-5}$
$\Gamma(\rho^\pm \rightarrow \pi^\pm \pi^+ \pi^-)$	$0.141 \pm 0.071$	$(9.32 \pm 4.71) \times 10^{-7}$
$\Gamma(K^{*-} \rightarrow \bar{K}^0 \pi^0 \pi^-)$	$11.5 \pm 0.5$	$(2.31 \pm 0.10) \times 10^{-4}$
$\Gamma(K^{*-} \rightarrow K^- \pi^+ \pi^-)$	$5.83 \pm 0.23$	$(1.17 \pm 0.05) \times 10^{-4}$
$\Gamma(K^{*-} \rightarrow K^- \pi^0 \pi^0)$	$0.593 \pm 0.025$	$(1.19 \pm 0.05) \times 10^{-5}$
$\Gamma(\bar{K}^{*0} \rightarrow K^- \pi^0 \pi^+)$	$15.9 \pm 0.6$	$(3.15 \pm 0.13) \times 10^{-4}$
$\Gamma(\bar{K}^{*0} \rightarrow \bar{K}^0 \pi^- \pi^+)$	$5.94 \pm 0.23$	$(1.18 \pm 0.05) \times 10^{-4}$
$\Gamma(\bar{K}^{*0} \rightarrow \bar{K}^0 \pi^0 \pi^0)$	$0.507 \pm 0.022$	$(1.00 \pm 0.04) \times 10^{-5}$